2021 24th International Conference on Computer and Information Technology (ICCIT), 18-20 December, 2021.

The Eigenvalue Distribution of Hankel Matrix: A Tool for Spectral Estimation From Noisy Data

Ahsanul Islam
Department of Electrical and Electronic Engineering
Khulna University of Engineering & Technology
Khulna, Bangladesh
ahsanulislamshuvo@gmail.com

Md. Rakibul Hasan
Department of Electrical and Electronic Engineering
BRAC University
Dhaka, Bangladesh
rakibul.hasan@bracu.ac.bd

Md Zakir Hossain
Biological Data Science Institute, Australian National University
and CSIRO A&F, Black Mountain
Canberra, Australia
zakir.hossain@anu.edu.au

Md. Mahbub Hasan
Department of Electrical and Electronic Engineering
Khulna University of Engineering & Technology
Khulna, Bangladesh
mahbub01@eee.kuet.ac.bd

2021 24th International Conference on Computer and Information Technology (ICCIT) | 978-1-6654-9435-9/21/$31.00 ©2021 IEEE | DOI: 10.1109/ICCIT54785.2021.9689845

***Abstract*—One of the key challenges of digital signal processing is to estimate sinusoidal components of an unknown signal. Researchers and engineers have been adopting various methods to analyze noisy signals and extract essential features of a given signal. Singular spectrum analysis (SSA) has been a popular and effective tool for extracting sinusoidal components of an unknown noisy signal. The process of singular spectrum analysis includes embedding time series into a Hankel matrix. The eigenvalue distribution of the Hankel matrix exhibits significant properties that can be used to estimate an unknown signal's rhythmic components and frequency response. This paper proposes a method that utilizes the Hankel matrix's eigenvalue distribution to estimate sinusoidal components from the frequency spectrum of a noisy signal. Firstly, an autoregressive (AR) model has been utilized for simulating time series employed to observe eigenvalue distributions and frequency spectrum. Nevertheless, the approach has been tested on real-life speech data to prove the applicability of the proposed mechanism on spectral estimation. Overall, results on both simulated and real data confirm the acceptability of the proposed method. This study suggests that eigenvalue distribution can be a helpful tool for estimating the frequency response of an unknown time series. Since the autoregressive model can be used to model various real-life data analyses, this study on eigenvalue distribution and frequency spectrum can be utilized in those real-life data. This approach will help estimate frequency response and identify rhythmic components of an unknown time series based on eigenvalue distribution.**

***Index Terms*—Hankel matrix, Eigenvalue distribution, Singular spectrum analysis, Autoregressive model, Frequency response**

I. Introduction

In this modern science and technology era, people are becoming increasingly dependent on electrical and electronic instruments. Nowadays, people are frequently using devices that can manipulate the digital signals received from various sensors. These digital signals often contain valuable information that can be used to serve various purposes. Most of these signals have a high tendency to get polluted by multiple sources of noise interference. This is where digital signal processing plays a significant role in extracting useful information from noisy data by minimizing the noise components. Nowadays, the task of digital signal processing has become more accessible to implement due to the availability of complex computational devices [1]. In the process of removing noise from a digital signal, singular spectrum analysis (SSA) can be used as a reliable tool [2]. Various fields of science are adopting this novel technique for numerous tasks such as time series analysis, multivariate statistics, signal processing, and similar applications [3]. The SSA is a non-parametric method of time series analysis that does not require any statistical assumption to analyze the data from noise components [3].

The first step of the SSA technique includes embedding the signal's time series into a trajectory matrix called Hankel Matrix [4]. Eigenvalue distribution of the Hankel matrix of a particular time series can be used to analyze the property of a signal. For instance, the distribution of the eigenvalues of the embedded Hankel matrix of a time series exhibits a specific pattern for a white noise process that can be utilized to filter out noise components of a signal [5]. Statistical parameters such as skewness and kurtosis can help select the optimum number of eigenvalues. It can be utilized in separating noise components from EEG signals [6]. The distribution of eigenvalues can also help in separating noise components from chaos [7]. The harmonic components of a signal produce a pair of singular values that are very close to each other [4]. It can help visualize through plots. Analyzing the different patterns and properties of eigenvalues can help assume the spectral property of a signal and separate the noisy elements.

The eigenvalue distribution can be used to extract the periodic components of different types of signals, which can then be reconstructed into signals with different rhythms. For example, extraction of rhythmic components of EEG can be done and then be reconstructed to signals with multiple rhythmic components with the aid of eigenvalue distribution [8]. It is theoretically significant to develop an algorithm based on the eigenvalue distribution to analyze the frequency spectrum and rhythmic elements of a signal, which can be employed in rhythm extraction of various unknown signals. Researchers have proposed several methods to denoise signals based on SSA method [9]–[12]. In SSA, the eigenvalues of the Hankel matrix contain essential information about the signal. In general cases, the eigenvalue distribution is observed to choose the optimum number of eigenvalues along with the eigenvectors in order to reconstruct the given signal avoiding noise components. This indicates that the noise components and the signal components can be identified with the help of eigenvalues and how it is distributed. It seems that the eigenvalue distribution can help estimate the sinusoidal components of a given signal. Therefore, it is possible to develop a method based on eigenvalue distribution that helps us get an idea about the system of an unknown signal.

This article proposes a method to estimate the sinusoidal components of a signal based on the eigenvalue distribution of the Hankel matrix. In practice, there can be many unknown signals. The proposed method can be utilized to have an estimation of the sinusoidal components of the unknown signal. In this article, the eigenvalue distribution of a simulated time series is observed, and its frequency spectrum is also shown. In addition, the dispersion of eigenvalues in the distribution and its relation to its corresponding frequency spectrum is also discussed. Furthermore, the proposed method has been applied to real-life speech data where the eigenvalue distribution and frequency spectrum of five speech sounds have been investigated. These properties of eigenvalue distribution and frequency spectrum can be helpful in the extraction of periodic components of a signal, which can be used in real-life data analysis.

II. Mathematical Formulation of SSA-Based Rhythm Estimation

The method of evaluating rhythms is discussed in this section. In particular, the process of SSA is briefly introduced, followed by a discussion on the dispersions of the eigenvalues. The process is used for decomposing a particular time series into a sum of trends, harmonics, and noise components [4]. This process aims to separate the noise components of the signal to extract authentic information provided through the signal.

Suppose a time series of length N be $\mathbf{x} = (x_1, x_2, \cdots, x_N)$. As shown in (1), the time series is embedded into a trajectory matrix $\mathbf{X}$ with a dimension of window length, L by K having $K = N - L + 1$.

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_K \\ x_2 & x_3 & x_4 & \cdots & x_{K+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_L & x_{L+1} & x_{L+2} & \cdots & x_N \end{bmatrix} \tag{1}$$

The matrix $\mathbf{X}$ is called the Hankel matrix, where all the anti-diagonal elements are equal. The method of singular value decomposition (SVD) is then performed to this Hankel Matrix. The first step of SVD is to construct the covariance matrix, $C = \mathbf{X}\mathbf{X}^T$. The covariance matrix of size L by L is then used to calculate the eigenvalues for further analysis. The eigenvalues are calculated using (2).

$$|C - \lambda I| = 0 \tag{2}$$

The calculated eigenvalues are denoted using the symbol λ_i, where $(i = 1, 2, 3, \cdots, L)$. The eigenvalues are sorted in descending order $(\lambda_1 \geq \lambda_2 \geq \cdots \lambda_L \geq \cdots \geq 0)$. The corresponding eigenvectors are $U_1, U_2, \cdots, U_L$. After performing SVD, the trajectory matrix $\mathbf{X}$ can be written according to (3).

$$\mathbf{X} = X_1 + X_2 + X_3 + \cdots + X_d \tag{3}$$

Here, d is the rank of matrix $\mathbf{X}$. If $V_i = X^T U_i / \sqrt{\lambda_i}$, then according to (3), $\mathbf{X}_i = \sqrt{\lambda_i} U_i V_i^T$ can be written. After that, grouping is performed using several criteria, which is then followed by diagonal averaging to reconstruct the time series to mitigate the noise component.

Reconstructed components X_1 and X_2 will form sinusoidal rhythm if the eigenvalues are equal or close. In this article, the deviations of the eigenvalue pairs and the sinusoidal components are investigated. The Fourier transform tells about the frequency spectrum of the whole signal, including noise-spectrum, yet it does not tell us about the system-generated sinusoidal component. The eigenvalue distribution will help identify the system-induced important sinusoidal component within the signal. In order to get an estimation of the sinusoidal components of a given signal, the first step is to calculate the eigenvalues of the Hankel matrix. After that, the eigenvalues need to be plotted so that the distribution of eigenvalues can be visualized. In the distribution of eigenvalues, the harmonic components produce a pair of eigenvalues that remains very close to each other [4]. Therefore, the number of sinusoidal components can be estimated by observing the number of pairs in the eigenvalue distribution. Each pair of close eigenvalues is responsible for one sinusoidal component in the signal. The magnitude of the pair of eigenvalues depends on the strength of the sinusoidal component.

III. Computer Generated Signals

In this paper, an autoregressive model is used for signal generation. The eigenvalue distribution of that time series is observed and then compared with its frequency spectrum. An autoregressive model can be used to model different types of signals along with EEG signal [13], [14] and financial time series [15]. The second-order autoregressive model (AR(2)) is used to generate time series for simulation purposes, for which the equation is given in (4).

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \epsilon_t \tag{4}$$

where, ϕ_1 and ϕ_2 represent the AR(2) model parameters, and ϵ_t represents input random noise parameter. In the simulation, 512 time-samples are generated as per the AR(2) model given by (4). One of the generated time series is depicted in Fig. 1.

In the simulation, input noise variance is kept constant at 0.1, and the other parameters ϕ_1 and ϕ_2 are varied. For

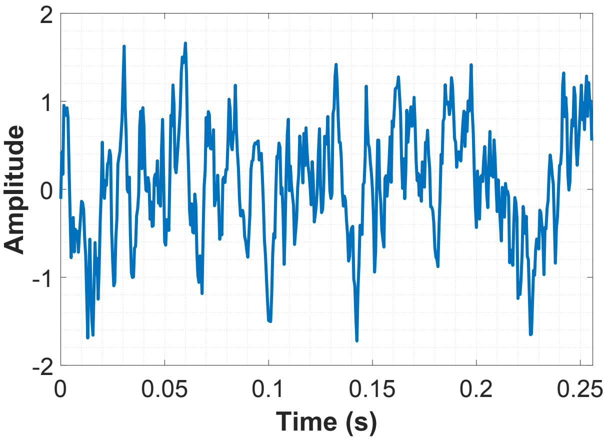


Fig. 1. Sample time series simulated from AR(2) model.

different values of ϕ_1 and ϕ_2, the corresponding eigenvalue distributions and frequency spectrums are observed.

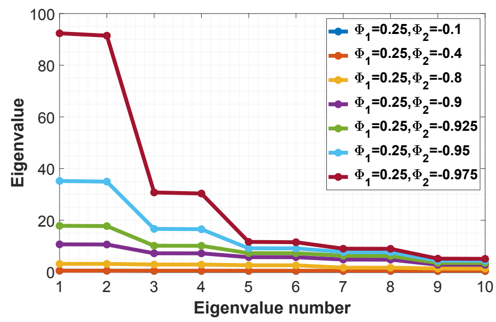
IV. Implementation of the Proposed Method

A. Eigenvalue Distribution

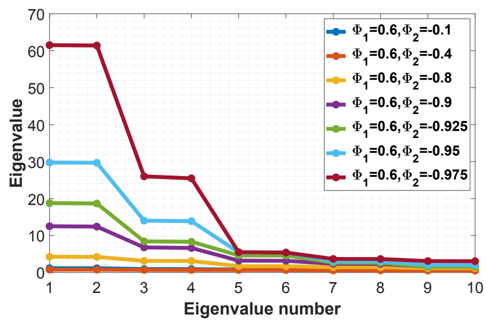
The generated time series is embedded into a Hankel matrix with a window length of $L = 180$. After that, the covariance matrix is obtained by multiplying the Hankel matrix with its transpose matrix. This covariance matrix leads towards the calculation of the eigenvalues. For different values of ϕ_1 and ϕ_2 for the time series, the corresponding eigenvalues are also calculated. In Fig. 2, the first ten eigenvalues of the Hankel matrix are plotted for multiple values of the AR(2) model parameters ϕ_1 and ϕ_2.

In Fig. 2a, it is seen that as the value of ϕ_2 gets closer to -1, the first four eigenvalues tend to have more and more dispersion from the last eigenvalues. Especially, the first two eigenvalues tend to dominate over the whole distribution concerning their amplitude. It is also observed from the distribution that the first two and second two eigenvalues are very close to each other. This property of making close pairs is seen for all the cases in Fig. 2. It makes an indication that the time series contains harmonic components [4]. It is observed that for comparatively smaller AR(2) parameter values, the amplitude of the first two eigenvalues gets lower, and their dispersion from the other eigenvalues also gets lower. This particular property is seen in all subfigures of Fig. 2. In Figs. 2a and 2b, the dispersion of the first two eigenvalues are pretty similar, and the third and fourth eigenvalue also seems to have a decent amplitude compared to other eigenvalues. In Figs. 2c and 2d, the first two eigenvalues possess quite a similar sort of dominating characteristics in the distribution; on the other hand, the third and fourth eigenvalues get lower. Furthermore, the dispersions of those third and fourth eigenvalues from the first two get higher and get closer to the last eigenvalues.

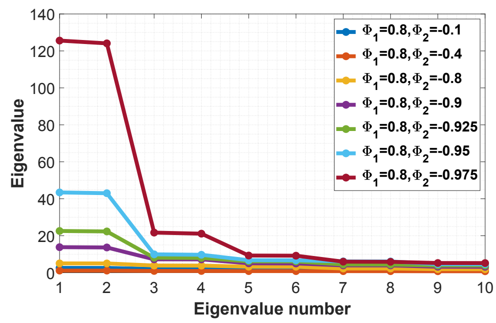
Here it is also observed in Fig. 2 that the last eigenvalues tend to get closer to zero regardless of the parameter values of the AR(2) model time series. Additionally, the last eigenvalues



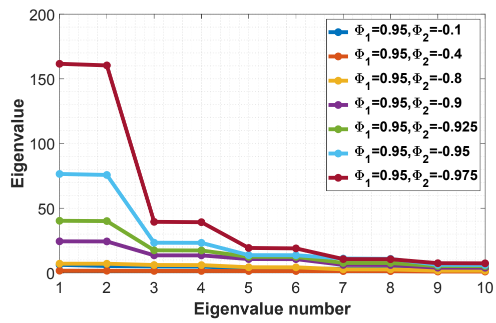
(a)



(b)



(c)

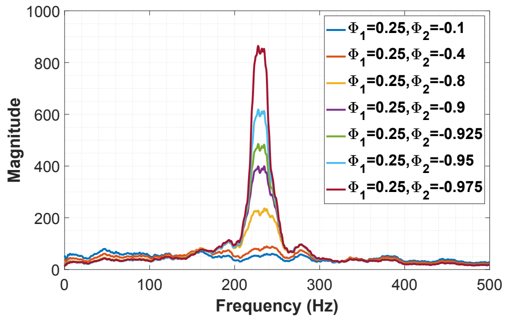


(d)

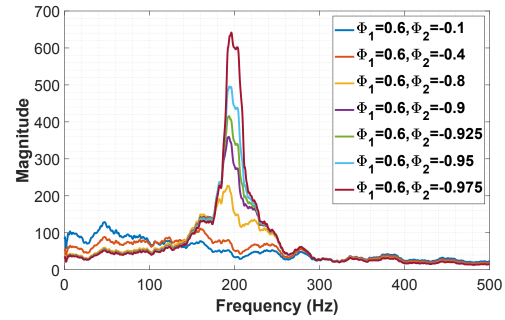
Fig. 2. Eigenvalue distribution for (a) $\phi_1 = 0.25$, (b) $\phi_1 = 0.6$, (c) $\phi_1 = 0.8$, and (d) $\phi_1 = 0.95$ with multiple values of ϕ_2.

exhibit closer to zero in the whole distribution of the Hankel matrix's eigenvalues. In general cases of the SSA method, the eigenvalues with higher and similar values are preferred for grouping and reconstruction. Researchers have proposed several ways of selecting the optimum number of eigenvalues for some specific types of time series [4].

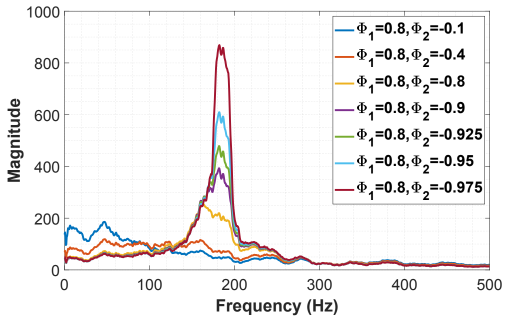
B. Frequency Spectrum and Comparison With Eigenvalue Distribution



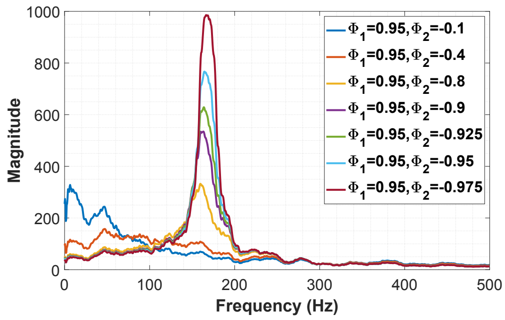
(a)



(b)



(c)



(d)

Fig. 3. Frequency spectrum for (a) $\phi_1 = 0.25$, (b) $\phi_1 = 0.6$, (c) $\phi_1 = 0.8$, and (d) $\phi_1 = 0.95$ with multiple values of ϕ_2.

The frequency spectrum of the simulated time series is plotted in Fig. 3. The frequency spectrum is also observed for multiple values of ϕ_1 and ϕ_2. In Fig. 3a, it is seen that the spectrum tends to have stronger peaks as the value of ϕ_2 gets closer to -1. It is also noticed from the plot that the most dominant peak of the frequency has the same parameter value for which the eigenvalue distribution is most dominant in the eigenvalue distribution plot shown in Fig. 2a. In Fig. 3a, the value of ϕ_2 is varied keeping ϕ_1 fixed at 0.25. In that plot, the frequency spectrum exhibits the strongest peak for $\phi_2 = -0.975$, and as the absolute value of ϕ_2 gets lower and closer to 0, the peak gets lower in that similar order. In the eigenvalue distribution plot shown in Fig. 2a, the pair of the first two eigenvalues get the highest value compared to others when $\phi_2 = -0.975$, and as the absolute value of ϕ_2 gets closer to zero, the values of these first two eigenvalues also get lower in the same order. In Fig. 3b, the frequency spectrum is similarly plotted for several values of ϕ_2 where ϕ_1 is kept constant at 0.6. Here in this frequency spectrum, the peaks of the frequency response get weaker than that of Fig. 3a. Apart from that, the strength of frequency response peak shows a similar sort of correlation with the dominance property of the first two eigenvalues shown in Fig. 2b. This particular correlation of frequency response with the dominance of the first two eigenvalues is also seen in between Figs. 3c, 3d and Figs. 2c, 2d. Therefore, after observing all these plots in Fig. 3 and Fig. 2, it is remarked that frequency response with stronger peaks tends to show dominating value for the first two eigenvalues in the eigenvalue distribution.

The time series having a comparatively weaker frequency response tends to get lower values for the first two eigenvalues in the whole eigenvalue distribution. As an example, the time series generated from AR(2) model with $\phi_2 = -0.1$ has got a weaker frequency response in the frequency spectrum as shown in Fig. 3. The first two eigenvalues of the same time series got lower values with lower dominance over the distribution of the eigenvalues. Therefore, it is observed that the time series in which the first few eigenvalues tend to show dominance over the eigenvalue distribution have stronger peaks in the frequency spectrum. This particular property of eigenvalues can assist in estimating the frequency response of a given time series. In Figs. 3a and 3b, it seems that some signals tend to have two sinusoidal peaks in the frequency response. In Figs. 2a and 2b, it is also observed that the pair of third and fourth eigenvalues seems to have a decent magnitude compared to others, which is why there can be seen two peaks for some signals in Figs. 3a and 3b. The eigenvalue distribution of the Hankel matrix time series can help in estimating whether a signal has strong sinusoidal components or not. In practice, the property of a signal is usually unknown. The Hankel matrix's eigenvalue distribution can be a helpful tool to estimate the frequency response, which can help understand the signal.

V. Investigation Into Speech Signals

To test the proposed method on real-life signals, five sample speech signals are utilized from the dataset presented in [16]. These sounds are Bengali language's first vowel equivalent to the /ɔ/ sound in English, one of which is depicted in Fig. 4.

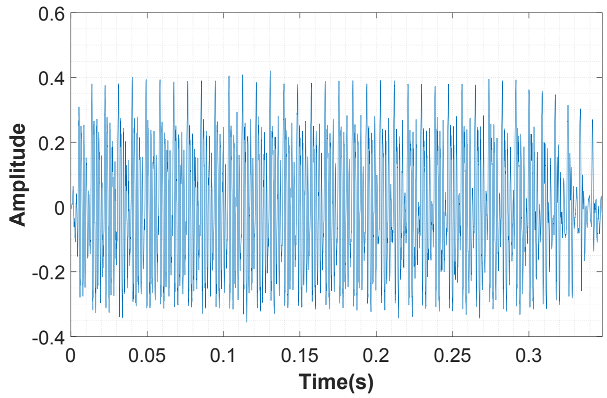


Fig. 4. Waveshape of a real-life signal (the first vowel sound of Bengali language).

Since the sounds were recorded in a conventional smartphone without any noise suppressing environment, some noise was inherently embedded within the signals. Both eigenvalue distribution and frequency spectrum for all five sample sounds have been produced, and they are presented in Fig. 5.

Now it is time to correlate the eigenvalue pairs with the number of the frequency components. However, since there are noises within the signals, some eigenvalue pairs having smaller values correspond to those noises. Hence, the question arises—how many eigenvalue pairs to consider, and how many frequency peaks to take into account as actual components rather than noise? To address this issue, The following empirical rules are utilized:

i. From the frequency spectrum, identify the minimum considerable magnitude, M_{min}.
ii. Identify the maximum magnitude (M_{max}) in the spectrum.
iii. Calculate the ratio, $r = (M_{min}/M_{max})^2$. Here, squaring is performed to consider energy since absolute value was considered while plotting the frequency spectrum.
iv. Count the number of frequency peaks (N_{freq}) between M_{min} and M_{max}.
v. From the eigenvalue distribution, find the peak of the largest eigenvalue pairs, and take the average of these two eigenvalues, E_{max}.
vi. Consider the minimum threshold to count eigenvalue pairs according to $E_{min} = r \times E_{max}$.
vii. Count the number of eigenvalue pairs (N_{eigen}) between E_{min} and E_{max} along the eigenvalue axis.

The above technique helps select eigenvalue pairs according to the energy ratio of the maximum peak to the considerable minimum frequency peak. Here the maximum and minimum frequency peak denotes the magnitudes of frequency in the spectrum plot, and, of course, it does not indicate the frequency value itself. The rules impose that the eigenvalue ratio E_{min}/E_{max} equals to the frequency spectrum ratio r. According to the proposed approach, the number of spectral peaks (N_{freq}) and the number of eigenvalue pairs (N_{eigen}) should be equal to each other. The above rules are applied to all five samples, and the results are tabulated in Table I.

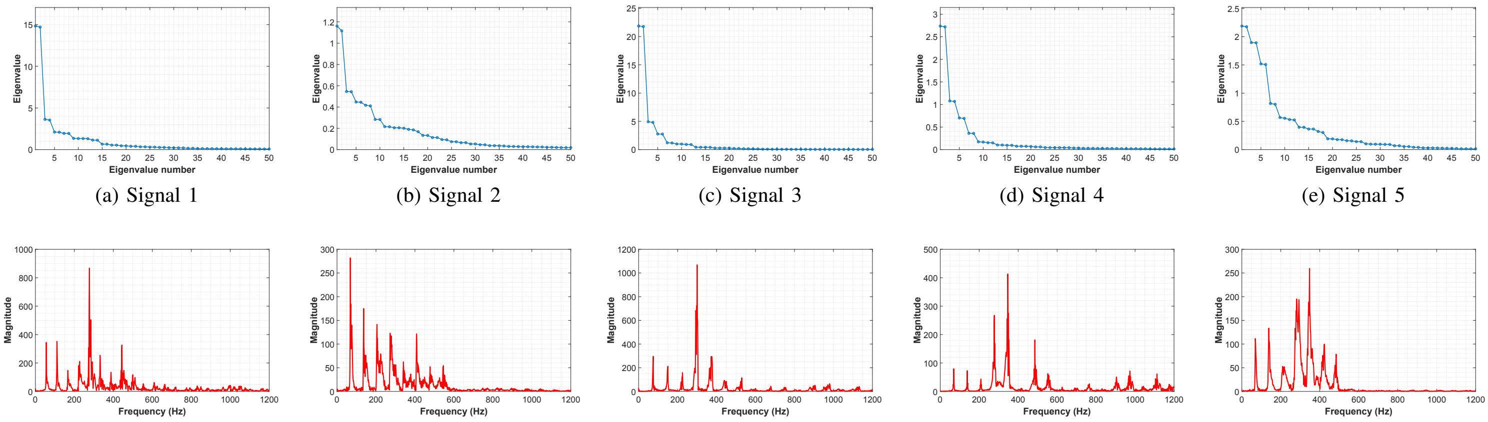
In the frequency spectrum of Signal 1 shown in Fig. 5a, the lowest magnitude considered as an original signal component is 253.1. All other frequencies having magnitude below this level are considered as noise/unwanted signal components. The highest peak is 867.4, and thus the squared ratio becomes 0.0851. Counting the number of frequency peaks between the magnitude of 253.1 and 867.4 results in six peaks. From the eigenvalue spectrum of Signal 1 shown in Fig. 5f, the average of the largest eigenvalue is 14.7677. Multiplying with the spectrum ratio of 0.0851, the lower threshold to count eigenvalue pairs becomes 1.2574. Accordingly, the number of eigenvalue pairs between 1.2574 and 14.7677 is counted. As expected, there are six eigenvalue pairs within this range, which is exactly the same as the number of the frequency peaks.

TABLE I
Test on five real-life signals. It indicates that the number of eigenvalue pairs reflects the number of frequency peaks for all five samples.

Signal	M_{min}	M_{max}	r	N_{freq}	σ_{freq}	E_{max}	E_{min}	N_{eigen}	σ_{eigen}
1	253.1	867.4	0.0851	6	229.88	14.7677	1.2574	6	5.0120
2	121.5	281.7	0.1860	6	60.94	1.1380	0.2117	6	0.3156
3	295.5	1068	0.0766	3	386.25	21.8200	1.6704	3	9.3461
4	181.2	413.1	0.1924	3	117.25	2.7291	0.5251	3	0.9672
5	99.71	259.4	0.1478	8	77.09	2.1826	0.3225	8	0.6997

M_{min}: Minimum considerable magnitude in the frequency spectrum
M_{max}: Maximum magnitude in the frequency spectrum
$r = (M_{min}/M_{max})^2$: Squared spectrum ratio
N_{freq}: Counted number of frequency peaks within the range
σ_{freq}: Standard deviation of the magnitudes of the frequency peaks
E_{max}: Average of the two largest eigenvalues
$E_{min} = r \times E_{max}$: Minimum threshold to count eigenvalue pairs
N_{eigen}: Counted number of eigenvalue pairs within the calculated range
σ_{eigen}: Standard deviation of the averages of each eigenvalue pairs

Other samples of the table also follow the same pattern—the number of frequency peaks and the number of eigenvalue pairs are equal to each other. The standard deviation for both the magnitude of the considered frequency peaks (σ_{freq}) and the eigenvalue pairs (σ_{eigen}) have been calculated. The average for each pair has been calculated, and then the standard deviation has been obtained among these averages. It is worthwhile to note that the correlation coefficient between these two standard deviations $(\sigma_{freq}, \sigma_{eigen})$ is 0.9951, which means they are strongly correlated. Therefore, the distribution of the considered frequency peaks and the eigenvalue distribution are strongly linked. The higher eigenvalue pairs stand for higher magnitudes, and similarly, the lower eigenvalue pairs stand for the lower magnitudes in the frequency spectrum. This analysis on real-life signals complies with the conclusions made on simulated time-series signals (see Section IV-B). Therefore,



(a) Signal 1 (b) Signal 2 (c) Signal 3 (d) Signal 4 (e) Signal 5

(f) Signal 1 (g) Signal 2 (h) Signal 3 (i) Signal 4 (j) Signal 5

Fig. 5. Eigenvalue distribution and frequency spectrum of all five speech sounds.

the frequency component can be estimated from the eigenvalue distribution, and they are strongly correlated.

VI. Conclusion

This article inspects frequency spectrum and eigenvalue distribution of a Hankel matrix for both computer-generated and five real-life speech signals. Besides, dispersions of eigenvalues for the computer-generated signal have been examined and observed its' effect on the frequency spectrum. The results confirm that frequency response with stronger peaks tends to show higher domination of the first pair of eigenvalues in eigenvalue distribution. This property indicates that a time-series system with stronger sinusoidal components tends to show an eigenvalue distribution where the first few eigenvalue pairs have dominating values compared to other ones. Accordingly, this particular property can be a handy tool to identify whether a signal has sinusoidal components or not. Investigation on speech signals verifies that the proposed method is equally applicable for real-life data analysis. It has been further observed that the standard deviations of considered eigenvalues and frequency peaks' magnitude in the frequency spectrum are highly correlated to each other. This indicates that the eigenvalue distribution of a signal is strongly linked with the frequency spectrum. This particular property can be a helpful tool in identifying the system of an unknown signal. Particularly, detection of communication channels, estimation of frequency components in EEG, EMG, ECG, speech signals, and others can be done based on the eigenvalue distribution. Therefore, this particular property can be applied to noisy real-life data to estimate an unknown signal's rhythmic components and frequency response.

References

[1] J. G. Proakis and D. G. Manolakis, *Digital Signal Processing : Principles, Algorithms, and Applications*, 3rd ed. USA: Prentice-Hall, Inc., 1996.

[2] N. Alharbi and H. Hassani, "A new approach for selecting the number of the eigenvalues in singular spectrum analysis," *Journal of the Franklin Institute*, vol. 353, no. 1, pp. 1–16, 2016.

[3] N. Golyandina, V. Nekrutkin, and A. A. Zhigljavsky, *Analysis of time series structure: SSA and related techniques*. CRC press, 2001.

[4] S. Sanei and H. Hassani, *Singular spectrum analysis of biomedical signals*. CRC press, 2015.

[5] M. Ghodsi, N. Alharbi, and H. Hassani, "The empirical distribution of the singular values of a random hankel matrix," *Fluctuation and Noise Letters*, vol. 14, no. 03, p. 1550027, 2015.

[6] R. Ingle and R. Awale, "A unique approach of noise elimination from electroencephalography signals between normal and meditation state," *International Journal of Engineering*, vol. 31, no. 5, pp. 719–728, 2018.

[7] H. Hassani, N. Alharbi, and M. Ghodsi, "Distinguishing chaos from noise: A new approach," *International Journal of Energy and statistics*, vol. 2, no. 02, pp. 137–150, 2014.

[8] H. Hu, S. Guo, R. Liu, and P. Wang, "An adaptive singular spectrum analysis method for extracting brain rhythms of electroencephalography," *PeerJ*, vol. 5, p. e3474, 2017.

[9] Z. Hou, Y. Dong, and X. Wu, "A template addition method for eigentriple rearrangement in singular spectrum analysis for processing biopotential signals with extremely lower snrs," *IEEE Sensors Journal*, vol. 20, no. 6, pp. 3142–3150, 2019.

[10] Q. Liu, A. Liu, X. Zhang, X. Chen, R. Qian, and X. Chen, "Removal of emg artifacts from multichannel eeg signals using combined singular spectrum analysis and canonical correlation analysis," *Journal of Healthcare Engineering*, vol. 2019, 2019.

[11] A. K. Maddirala and R. A. Shaik, "Motion artifact removal from single channel electroencephalogram signals using singular spectrum analysis," *Biomedical Signal Processing and Control*, vol. 30, pp. 79–85, 2016.

[12] S. M. Mohammadi, S. Kouchaki, M. Ghavami, and S. Sanei, "Improving time–frequency domain sleep eeg classification via singular spectrum analysis," *Journal of neuroscience methods*, vol. 273, pp. 96–106, 2016.

[13] H.-W. Steinberg, T. Gasser, and J. Franke, "Fitting autoregressive models to eeg time series: An empirical comparison of estimates of the order," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 33, no. 1, pp. 143–150, 1985.

[14] F. Vaz, P. G. de Oliveira, and J. Principe, "A study on the best order for autoregressive eeg modelling," *International journal of bio-medical computing*, vol. 20, no. 1-2, pp. 41–50, 1987.

[15] C. Vidal and A. Suárez, "Hierarchical mixtures of autoregressive models for time-series modeling," in *Artificial Neural Networks and Neural Information Processing — ICANN/ICONIP 2003*, O. Kaynak, E. Alpaydin, E. Oja, and L. Xu, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2003, pp. 597–604.

[16] M. R. Hasan and M. M. Hasan, "Investigation of the effect of mfcc variation on the convolutional neural network-based speech classification," in *2020 IEEE Region 10 Symposium (TENSYMP)*, 2020, pp. 1408–1411.